Significant Figures

By the same author

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Lives and Works of Trailblazing Mathematicians

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Introduction

ALL BRANCHES OF SCIENCE can trace their origins far back into the mists of history, but in most subjects the history is qualified by 'we now know this was wrong' or 'this was along the right lines, but today's view is different'. For example, the Greek philosopher Aristotle thought that a trotting horse can never be entirely off the ground, which Eadweard Muybridge disproved in 1878 using a line of cameras linked to tripwires. Aristotle's theories of motion were completely overturned by Galileo Galilei and Isaac Newton, and his theories of the mind bear no useful relation to modern neuroscience and psychology.

Mathematics is different. It endures. When the ancient Babylonians worked out how to solve quadratic equations – probably around 2000 BC, although the earliest tangible evidence dates from 1500 BC – their result never became obsolete. It was correct, and they knew why. It's still correct today. We express the result symbolically, but the reasoning is identical. There's an unbroken line of mathematical thought that goes all the way back from tomorrow to Babylon. When Archimedes worked out the volume of a sphere, he didn't use algebraic symbols, and he didn't think of a specific *number* π as we now do. He expressed the result geometrically, in terms of proportions, as was Greek practice then. Nevertheless, his answer is instantly recognisable as being equivalent to today's $^4/_3\pi r^3$.

To be sure, a few ancient discoveries outside mathematics have been similarly long-lived. Archimedes's Principle that an object displaces its own weight of liquid is one, and his law of the lever is another. Some parts of Greek physics and engineering live on too. But in those subjects, longevity is the exception, whereas in mathematics it's closer to the rule. Euclid's *Elements*, laying out a logical basis for geometry, still repays close examination. Its theorems remain true, and many remain useful. In mathematics, we move on, but we don't discard our history.

Before you all start to think that mathematics is burying its head in the past, I need to point out two things. One is that the perceived importance of a method or a theorem can change. Entire areas of mathematics have gone out of fashion, or become obsolete as the frontiers shifted or new techniques took over. But they're still *true*, and from time to time an obsolete area has undergone a revival, usually because of a newly discovered connection with another area, a new application, or a breakthrough in methodology. The second is that as mathematicians have developed their subject, they've not only moved on; they've also devised a gigantic amount of new, important beautiful, and useful mathematics.

That said, the basic point remains unchallenged: once a mathematical theorem has been correctly proved, it becomes something that we can build on – forever. Even though our concept of proof has tightened up considerably since Euclid's day, to get rid of unstated assumptions, we can fill in what we now see as gaps, and the results still stand.

Significant Figures investigates the almost mystical process that brings new mathematics into being. Mathematics doesn't arise in a vacuum: it's created by *people*. Among them are some with astonishing originality and clarity of mind, the people we associate with great breakthroughs – the pioneers, the trailblazers, the significant figures. Historians rightly explain that the work of the greats depended on a vast supporting cast, contributing tiny bits and pieces to the overall puzzle. Important or fruitful questions can be stated by relative unknowns; major ideas can be dimly perceived by people who lack the technical ability to turn them into powerful new methods and viewpoints. Newton remarked that he 'stood on the shoulders of giants'. He was to some extent being sarcastic; several of those giants (notably Robert Hooke) were complaining that Newton was not so much standing on their shoulders as treading on their toes, by not giving them fair credit, or by taking the credit in public despite citing their contributions in his writings. However, Newton spoke truly: his great syntheses of motion, gravity, and light depended on a huge number of insights from his intellectual predecessors. Nor were they exclusively giants. Ordinary people played a significant part too.

Nevertheless, the giants stand out, leading the way while the rest of us follow. Through the lives and works of a selection of significant figures, we can gain insight into how new mathematics is created, who created it, and how they lived. I think of them not just as pioneers who showed the rest of us the way, but as trailblazers who hacked traversable paths through the tangled undergrowth of the sprawling jungle of mathematical thought. They spent much of their time struggling through thorn bushes and swamps, but from time to time they came across a Lost City of the Elephants or an El Dorado, uncovering precious jewels hidden among the undergrowth. They penetrated regions of thought previously unknown to humankind.

Indeed, they *created* those regions. The mathematical jungle isn't like the Amazon Rainforest or the African Congo. The mathematical trailblazer isn't a David Livingstone, hacking a route along the Zambezi or hunting for the source of the Nile. Livingstone was 'discovering' things that were *already there*. Indeed, the local inhabitants knew they were there. But in those days, Europeans interpreted 'discovery' as 'Europeans bringing things to the attention of other Europeans.' Mathematical trailblazers don't merely explore a pre-existing jungle. There's a sense in which they create the jungle as they proceed; as if new plants are springing to life in their footsteps, rapidly becoming saplings, then mighty trees. However, it *feels* as if there's a pre-existing jungle, because you don't get to choose which plants spring to life. You choose where to tread, but you can't decide to 'discover' a clump of mahogany trees if what actually turns up there is a mangrove swamp.

This, I think, is the source of the still popular Platonist view of mathematical ideas: that mathematical truths 'really' exist, but they do so in an ideal form in some sort of parallel reality, which has always existed and always will. In this view, when we prove a new theorem we just find out what has been there all along. I don't think that Platonism makes literal sense, but it accurately describes the process of mathematical research. You don't get to choose: all you can do is shake the bushes and see if anything drops out. In *What is Mathematics, Really?* Reuben Hersh offers a more realistic view of mathematics: it's a shared human mental construct. In this respect it's much like money. Money isn't 'really' lumps of metal or pieces of paper or numbers in a computer; it's a shared set of conventions about how we exchange lumps of metal, pieces of paper, and numbers in a computer, for each other or for goods.

Hersh outraged some mathematicians, who zoomed in on 'human construct' and complained that mathematics is by no means arbitrary. Social relativism doesn't hack it. This is true, but Hersh explained perfectly clearly that mathematics isn't *any* human construct. We choose to tackle Fermat's Last Theorem, but we don't get to choose whether it's true or false. The human construct that is mathematics is subject to a stringent system of logical constraints, and something gets added to the construct only if it respects those constraints. Potentially, the constraints allow us to distinguish truth from falsity, but we don't find out which of those applies by declaiming loudly that only one of them is possible. The big question is: which one? I've lost count of the number of times someone has attacked some controversial piece of mathematics that they dislike by pointing out that mathematics is a tautology: everything new is a logical consequence of things we already know. Yes, it is. The new is implicit in the old. But the hard work comes when you want to make it explicit. Ask Andrew Wiles; it's no use telling him that the status of Fermat's Last Theorem was always predetermined by the logical structure of mathematics. He spent seven years finding out what its predetermined status *is*. Until you do that, being predetermined is as useful as asking someone the way to the British Library and being told that it's in Britain.

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Significant Figures isn't an organised history of the whole of mathematics, but I've tried to present the mathematical topics that arise in a coherent manner, so that the concepts build up systematically as the book proceeds. On the whole, this requires presenting everything in roughly chronological order. Chronological order by topic would be unreadable, because we'd be perpetually hopping from one mathematician to another, so I've ordered the chapters by birth date and provided occasional cross-references.[1]

My significant figures are 25 in number, ancient and modern, male and female, eastern and western. Their personal histories begin in ancient Greece, with the great geometer and engineer Archimedes, whose achievements ranged from approximating π and calculating the area and volume of a sphere, to the Archimedean screw for raising water and a crane-like machine for destroying enemy ships. Next come three representatives of the far east, where the main mathematical action of the Middle Ages took place: the Chinese scholar Liu Hui, the Persian mathematician Muhammad ibn Musa Al-Khwarizmi, whose works gave us the words 'algorithm' and 'algebra', and the Indian Madhava of Sangamagrama, who pioneered infinite series for trigonometric functions, rediscovered in the west by Newton a millennium later.

The main action in mathematics returned to Europe during the Italian Renaissance, where we encounter Girolamo Cardano, one of the biggest rogues ever to grace the mathematical pantheon. A gambler and brawler, Cardano also wrote one of the most important algebra texts ever printed, practised medicine, and led a life straight out of the tabloid press. He cast horoscopes, too. In contrast, Pierre de Fermat, famous for his Last Theorem, was a lawyer with a passion for mathematics that often led him to neglect his legal work. He turned number theory into a recognised branch of mathematics, but also contributed to optics and developed some precursors to calculus. That subject was brought to fruition by Newton, whose masterwork is his *Philosophiae Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy), usually abbreviated to *Principia*. In it, he stated his laws of motion and gravity, and applied them to the motion of the solar system. Newton marks a tipping point in mathematical physics, turning it into an organised mathematical study of what he called the 'System of the World'.

For a century after Newton, the focus of mathematics shifted to continental Europe and Russia. Leonhard Euler, the most prolific mathematician in history, turned out important papers at a journalistic rate, while systematising many areas of mathematics in a series of elegant, clearly written textbooks. No field of mathematics evaded his scrutiny. Euler even anticipated some of the ideas of Joseph Fourier, whose investigation of the transmission of heat led to one of the most important techniques in the modern engineer's handbook: Fourier analysis, which represents a periodic waveform in terms of the basic trigonometric functions 'sine' and 'cosine'. Fourier was also the first to understand that the atmosphere plays an important role in the Earth's heat balance.

Mathematics enters the modern era with the peerless researches of Carl Friedrich Gauss, a strong contender for the greatest mathematician of all time. Gauss began in number theory, sealed his reputation in celestial mechanics by predicting the reappearance of the newly discovered asteroid Ceres, and made major advances regarding complex numbers, least-squares data fitting, and non-Euclidean geometry, though he published nothing on the latter because he feared it was too far ahead of its time and would attract ridicule. Nikolai Ivanovich Lobachevsky was less diffident, and published extensively on an alternative geometry to that of Euclid, now called hyperbolic geometry. He and Janós Bolyai are now recognised as the rightful founders of non-Euclidean geometry, which can be interpreted as the natural geometry of a surface with constant curvature. Gauss was basically right to believe that the idea was ahead of its time, however, and neither Lobachevsky nor Bolyai was appreciated during his lifetime. We round off this era with the tragic story of the revolutionary Évariste Galois, killed at the age of twenty in a duel over a young woman. He made major advances in algebra, leading to today's characterisation of the vital concept of symmetry in terms of transformation groups.

A new theme now enters the story, a trail blazed by the first female mathematician we encounter. Namely, the mathematics of computation. Augusta Ada King, Countess of Lovelace, acted as assistant to Charles Babbage, a single-minded individual who understood the potential power of calculating machines. He envisaged the Analytical Engine, a programmable computer made of ratchets and cogwheels, now the central gimmick of steampunk science fiction. Ada is widely credited with being the first computer programmer, although that claim is controversial. The computer theme continues with George Boole, whose *Laws of Thought* laid down a fundamental mathematical formalism for the digital logic of today's computers.

As mathematics becomes more diverse, so does our tale, hacking its way into new regions of the ever-growing jungle. Bernhard

Riemann was brilliant at uncovering simple, general ideas behind apparently complex concepts. His contributions include the foundations of geometry, especially the curved 'manifolds' upon which Albert Einstein's revolutionary theory of gravitation, General Relativity, depends. But he also made huge steps in the theory of prime numbers by relating number theory to complex analysis through his 'zeta function'. The Riemann Hypothesis, about the zeros of this function, is one of the greatest and most important unsolved problems in the whole of mathematics, with a million-dollar prize for its solution.

Next comes Georg Cantor, who changed the way mathematicians think about the foundations of their subject by introducing set theory, and defined infinite analogues of the counting numbers 1, 2, 3, … , leading to the discovery that some infinities are bigger than others – in a rigorous, meaningful, and useful sense. Like many innovators, Cantor was misunderstood and ridiculed during his lifetime.

Our second woman mathematician now appears on the scene, the prodigiously talented Sofia Kovalevskaia. Her life was rather complicated, tied up with Russian revolutionary politics and the obstacles that male-dominated society placed in the path of brilliant female intellectuals. It's amazing that she accomplished anything in mathematics at all. In fact, she made remarkable discoveries in the solution of partial differential equations, the motion of a rigid body, the structure of the rings of Saturn, and the refraction of light by a crystal.

The story now gathers pace. Around the turn of the nineteenth century, one of the world's leading mathematicians was the Frenchman Henri Poincaré. An apparent eccentric, he was actually extremely shrewd. He recognised the importance of the nascent area of topology – 'rubber-sheet geometry' in which shapes can be distorted continuously – and extended it from two dimensions to three and beyond. He applied it to differential equations, studying the three-body problem for Newtonian gravitation. This led him to discover the possibility of deterministic chaos, apparently random behaviour in a non-random system. He also came close to discovering Special Relativity before Einstein did.

As a German counterpart to Poincaré we have David Hilbert, whose career divides into five distinct periods. First, he took up a line of thought that originated with Boole, about 'invariants' – algebraic expressions that remain the same despite changes in coordinates. He then developed a systematic treatment of core areas of number theory. After that, he revisited Euclid's axioms for geometry, found them wanting, and added extra ones to plug the logical gaps. Next, he moved into mathematical logic and foundations, initiating a programme to prove that mathematics can be placed on an axiomatic basis, and that this is both consistent (no logical deduction can lead to a contradiction) and complete (every statement can either be proved or disproved). Finally, he turned to mathematical physics, coming close to beating Einstein to General Relativity, and introducing the notion of a Hilbert space, central to quantum mechanics.

Emmy Noether is our third and final female mathematician, who lived at a time when the participation of women in academic matters was still frowned upon by most of the incumbent males. She began, like Hilbert, in invariant theory, and later worked with him as a colleague. Hilbert made strenuous attempts to smash the glass ceiling and secure her a permanent academic position, with partial success. Noether blazed the trail of abstract algebra, pioneering today's axiomatic structures such as groups, rings, and fields. She also proved a vital theorem relating the symmetries of laws of physics to conserved quantities, such as energy.

By now the story has moved into the twentieth century. To show that great mathematical ability is not confined to the educated classes of the western world, we follow the life and career of the self-taught Indian genius Srinivasa Ramanujan, who grew up in poverty. His uncanny ability to intuit strange but true formulas was rivalled, if at all, only by giants such as Euler and Carl Jacobi. Ramanujan's concept of proof was hazy, but he could find formulas that no one else would ever have dreamed of. His papers and notebooks are still being mined today for fresh ways of thinking.

Two mathematicians with a philosophical bent return us to the foundations of the subject and its relation to computation. One is Kurt Gödel, whose proof that any axiom system for arithmetic must be incomplete and undecidable demolished Hilbert's programme to prove the opposite. The other is Alan Turing, whose investigations into the abilities of a programmable computer led to a simpler and more natural proof of these results. He is, of course, famous for his codebreaking work at Bletchley Park during World War II. He also proposed the Turing test for artificial intelligence, and after the war he worked on patterns in animal markings. He was gay, and died in tragic and mysterious circumstances.

I decided not to include any living mathematicians, but to end with two recently deceased modern mathematicians: one pure and the other applied (but also unorthodox). The latter is Benoit Mandelbrot, widely known for his work on fractals, geometric shapes that have detailed structure on all scales of magnification. Fractals often model nature far better than traditional smooth surfaces such as spheres and cylinders. Although several other mathematicians worked on structures that we now see as fractal, Mandelbrot made a great leap forward by recognising their potential as models of the natural world. He wasn't a theorem-proof type of mathematician; instead, he had an intuitive visual grasp of geometry, which led him to see relationships and state conjectures. He was also a bit of a showman, and an energetic promoter of his ideas. That didn't endear him to some in the mathematical community, but you can't please everyone.

Finally, I've chosen a (pure) mathematician's mathematician, William Thurston. Thurston, too, had a deep intuitive grasp of geometry, in a broader and deeper sense than Mandelbrot. He could do theorem-proof mathematics with the best of them, though as his career advanced he tended to focus on the theorems and sketch the proofs. In particular he worked in topology, where he noticed an unexpected connection with non-Euclidean geometry. Eventually, this circle of ideas motivated Grigori Perelman to prove an elusive conjecture in topology, due to Poincaré. His methods also proved a more general conjecture of Thurston that provides unexpected insights into all three-dimensional manifolds.

In the final chapter, I'll pick up some of the threads that weave their way through the 25 stories of these astonishing individuals, and explore what they teach us about pioneering mathematicians – who they are, how they work, where they get their crazy ideas, what drives them to be mathematicians in the first place.

For now, however, I'd just like to add two warnings. The first is that I've necessarily been selective. There isn't enough space to provide comprehensive biographies, to survey everything that my trailblazers worked on, or to enter into fine details of how their ideas evolved and how they interacted with their colleagues. Instead, I've tried to offer a representative selection of their most important – or interesting – discoveries and concepts, with enough historical detail to paint a picture of them as people and locate them in their society. For some mathematicians of antiquity, even that has to be very sketchy, because few records about their lives (and often no original documents about their works) have survived.

The second is that the 25 mathematicians I've chosen are by no means the *only* significant figures in the development of mathematics. I made my choices for many reasons – the importance of the mathematics, the intrinsic interest of the area, the appeal of the human story, the historical period, diversity, and that elusive quality, 'balance'. If your favourite mathematician is omitted, the most likely reason is limited space, coupled with a wish to choose representatives that are widely distributed in the three-dimensional manifold whose coordinates are geography, historical period, and gender. I believe that everyone in the book fully deserves inclusion, although one or two may be controversial. I have no doubt at all that many others could have been selected with comparable justification.